The evolution of the topology of high-voltage electricity networks

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Abstract: The electricity network represents an example of an evolving complex system. The first local networks contained only a few nodes, but within several decades, they have evolved into a highly connected continental system. The growth of these networks was influenced by various factors such as economic, demographic, political and technological developments. In this paper, we analyse the growth of the French 400 kV electricity transmission network from its establishment in 1960 until the year 2000. We study the different topological characteristics that describe the intensity of the growth process, such as the number of nodes, the number of lines, the average node connectivity and the overall length of wires. We compare these results with several economic and demographic indicators in order to identify the factors which correlate with the growth rate of the electricity network. Apart from this, we evaluate how the topological efficiency and vulnerability measures (clustering coefficient, information centrality, betweenness centrality) evolve in the course of time. The decisions regarding the power grid topology are influenced by many, very often contradictory factors, such as costs, the size of the covered area, demand, fault tolerance, reliability and quality of service. Our results yield a deeper insight into the process of Critical Infrastructure (CI) construction.
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**Keywords:** electricity system; topology analysis; network growth.


**Biographical notes:** Lubos Buzna attained his PhD after studies at the University of Zilina, Slovakia. Since 2003, he has been working as a Research Assistant at the Faculty of Management Science and Informatics of the University of Zilina. At present, he is a Postdoctoral Researcher at ETH Zurich, Chair of Sociology in particular, of Modelling and Simulation. His research activities are focused on the application of optimisation methods to transportation systems, distribution logistics and critical infrastructures.

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Dirk Helbing has been a Full Professor and the Managing Director at the Institute for Transport & Economics. Since 2007, he fills the Chair of Sociology, in particular of Modelling and Simulation, at ETH Zurich. He has published more than 100 papers, including several contributions to journals *Nature*, *Science*, or *Review of Modern Physics*. He has also (co-)edited several books and proceedings of the international conferences on cooperative dynamics in socioeconomic and traffic systems.

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1 **Introduction**

In the last century, some infrastructures such as the electricity network, telecommunications network, food supply, public and cargo transport grew to reach international and intercontinental dimensions, changing our world into a globally interconnected system. At the same time, humans’ daily activities became more and more dependent on the failure-free operation of these infrastructures. This has motivated public authorities to initiate intensive research on the protection methods of critical infrastructures and to increase their robustness against deliberate attacks and internal failures. The electricity supply has been identified as one of the most important critical infrastructures for economic performance and social welfare (Birchmeier, 2007). Its criticality originates from the very strong dependencies of all the other infrastructures on the electricity supply.

Functional models play an important role in the analysis of power system vulnerability (Milano, 2005; Anghel *et al.*, 2007). However, these models require very detailed data and are limited in their capabilities to capture the very intricate dynamic behaviour of large systems in real time. This has recently motivated several studies in which complex systems are abstracted as graphs (Albert *et al.*, 2004; Rosato *et al.*, 2007; Holmgren, 2006; Casals *et al.*, 2007; Latora and Marchiori, 2005). Structural vulnerability is then assessed based on the graph topology, which is assumed to imply
the characteristic properties of the system. In this paper, we also study not only the properties of the contemporary power grid network topology. Adopting the established methodology, we also study the evolution of the French 400 kV network from its beginning in 1960 until the year 2000. So far, studies of the evolution of network topologies have been performed, for example, for scientific papers citation and phone call networks (Palla et al., 2007; Barabasi et al., 2001), innovation networks (Valverde et al., 2006) or the internet (Rosato and Tiriticco, 2004).

Our paper is organised as follows. In Section 2, we describe the analysed data and compare the network growth rate with selected demographic and economic indicators. In Section 3, the trends for selected structural features are discussed and Section 4 concludes our paper, summarising its results and implications.

2 The growth of the French transmission electricity network

The electricity supply network today is a hierarchical system that transmits electric energy from generating facilities to places where it is consumed. At the top level is a high-voltage transmission network which carries the energy over long distances. It is designed to sustain large loads. This transmission network has three main functionalities:

1. to transport energy across the country
2. to back up the systems of neighbouring countries
3. to exchange energy between countries over long distances.

The modern French transmission network consists of power lines which carry a very high voltage (220 kV) and extra-high voltage (400 kV) from the power generation facilities to regional substations, where transformers lower the voltage so it can be distributed to the consumers. In its early days, the transmission network backbone was based solely on 220 kV lines. It was only in the middle of the last century that also 380 kV lines were introduced in the system, eventually forming the new backbone that is normalised at 400 kV, while the 220 kV lines became, in turn, regional transmission lines.

The need for a more powerful transmission network was raised by the increasing electricity demand in both the industrial and domestic markets. The developments in technology enabled the construction of bigger power plants which could satisfy the growing demand. The new power plants, mostly nuclear power plants, were located further away from the main consumption points and required more powerful transmission lines that allow the transmission of over longer distances and with minimum power losses. Later on, these lines also enabled the connection of the French power network with the power networks of neighbouring countries, finally forming the European transmission system.

Since the laying of the first 380 kV lines in the 1960s, the extra-high voltage network has been continuously growing. In an attempt to give a quantitative description of this process, we have analysed the publicly available data that describe the evolution of the French 400 kV electricity network. The dataset consists of 21 snapshots that show the network topology in different years (from 1960 to 2000) in biannual time steps (see Figure 1).
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The network topology of the French 400 kV transmission network in selected years (see online version for colours)

The electricity network is represented as an undirected weighted graph $G$ that consists of a set of $N$ nodes and $M$ edges connecting pairs of nodes. The nodes of the graph are substations and the links represent the power lines. To facilitate the definition of the used measures, we describe the graph by an adjacency matrix $A$ ($N \times N$), where $a_{ij} = 1$ indicates that nodes $i$ and $j$ are connected by at least one link and $a_{ij} = 0$, otherwise. Each link is weighted by its length $w_{ij} > 0$. Parallel links are assumed to be of the same length and the number of links that connects nodes $i$ and $j$ is represented by $N_{ij}$.

Using the abstraction as a graph, we can extract quantitative information related to the network’s growth rate, such as the number of line intersections, the number of lines intersections (nodes) and the lines’ length. The results are depicted in Figure 2. In the first snapshot (1960), we find a disconnected network of only six power lines sized to 1488 l.u. (length units). Forty years later, the situation is quite different.

In the year 2000, the network covers most of the French territory and consists of 291 lines with 20831 l.u. The enlargement of the network over this period is far from being a simple linear process, as can be seen in Figure 2. All of the three parameters (nodes number, lines number and length) show a similar, slow linear growth until around 1975. From 1978, the growth rate is dramatically changed until it seems to reach some kind of saturation from 1990 onwards. The reasons for the change in behaviour are probably a mixture of many different factors and it is beyond the scope of this work to analyse them in detail. However, we have looked at several of the factors which we believed could have a large impact and have compared them to the findings above. Figure 3 presents the population growth, the Gross Domestic Product (GDP), the number of installed nuclear reactors and the electricity generation, consumption, export and import.2
Figure 2  The growth of the French 400 kV electricity network in terms of the number of nodes or links and the overall length of the lines (see online version for colours)

![Graph showing the growth of the French 400 kV electricity network](image)

Note: The inset shows the increase in the length of lines $\Delta l$ over the years.

Figure 3  The gross domestic product of France per inhabitant in EUR, the population growth, the number of operating nuclear reactors and the generated, consumed, imported and exported electricity for the studied time period (see online version for colours)

![Graph showing GDP, population, and operating reactors](image)
The main challenge for transmission networks is to keep the balance between generation and consumption. Therefore, the national policy in electricity generation and the trends in the consumption of energy also have a major influence on the development of the transmission network topology. France is known for having a large part of its energy produced by nuclear power plants. Currently, over 78% of French electricity is produced from nuclear energy. There is a clear correlation between the number of installed nuclear reactors and the transmission network’s size (see Figure 3). Except for recent years, where the network growth has saturated, the latter also correlates with the evolution of GDP per capita. The intensive building of new generation capacities and transmission power lines between 1975 and 1990 allowed France to become one of the largest electricity exporters in Europe.

3 The evolution of the network’s topological properties

While in the previous section, we addressed the growth rate of the French transmission network, in this section, we study the evolution of the network’s structural properties. There are many possible topology-related measures that are available for network analysis (Costa et al., 2007), among them node degree-related measures and centrality measures, which we apply to the previously mentioned data. The degree $k_i$ of node $i$ is the number of edges that are connected to that node. The graphs that represent real networks are usually classified by their degree distribution. The commonly reported cumulative node degree distribution of large-scale power grids that connect several countries (Union for the Coordination of Transmission of Electricity (UCTE) network, Nordic power grid,
Western USA power grid) is exponential (Albert et al., 2004; Holmgren, 2006; Casals et al., 2007; Amaral et al., 2000). However, this does not appear to be the case for the French network, as can be seen in the comparison with an exponential fit for the networks’ degree distribution in the year 2000 (see Figure 4b). The degree distribution in this year shows that medium connectivities ($k = 3$ and $k = 4$) are more frequent than expected for an exponential distribution. This is quite typical for large central Western European countries (Rosato et al., 2007).

**Figure 4** The node degree distribution (a) and the cumulative node degree distribution (b) for selected years (see online version for colours)

![Node degree distribution](image1)

![Cumulative node degree distribution](image2)

*Note: The dashed line is the best exponential fit of the data in 2000 $\alpha \exp(-k/\beta)$, with $\alpha = 3.077$ and $\beta = 1.27$.***
Another node degree-related property that is used for the classification of real networks is the small-world property which, according to Holmgren (2006), is a common characteristic of power grids. The small-world property is defined based on the comparison of the clustering $C_A$ and the average path length $l_A$ with a random graph of the same node number $N$ and the same average node degree $\langle k \rangle$. In Figure 5, we compare the network clustering $C_A = (1/N) \sum_i c_i^2$ and its average path length $l_A$ with the expected network clustering $C_G = 2M/(N(N-1))$ in an equivalent random graph of the average path length $l_G = \log(N)/\log(\langle k \rangle)$. If $l_A < l_G$ and $C_A > C_G$, then the graph shows the small-world property (Holmgren, 2006). Based on this criterion, the French power grid seems to have gained this property only in the last 15 years, when the network was more tightly interconnected. Another interesting quality of electrical power networks is its average degree of connectivity $\langle k \rangle$. Figure 6 depicts the relation between the number of nodes and the number of links for all 21 snapshots of network topology. The relation between the nodes and links maintains a constant value of $\langle k \rangle \approx 2.8$ throughout the years, even though the degree distribution itself varies over the years (see Figure 4a). A similar result was reported in Casals et al. (2007) for all of the contemporary topologies of UCTE members, showing a universal mean node degree of the same value. Note, however, that the measurements in Casals et al. (2007) were done for one configuration of the system, while we show that this property is time-independent.

Figure 5  (a) The average path length $l_A$ in unweighted graphs and the average path length in an equivalent random graph, calculated as $l_G = \log(N)/\log(\langle k \rangle)$ and (b) network clustering $C_A = (1/N) \sum_i c_i^2$ and the clustering component $C_G = 2M/(N(N-1))$ (see online version for colours)
Figure 5  (a) The average path length $l_A$ in unweighted graphs and the average path length in an equivalent random graph, calculated as $l_G = \log(N)/\log(k)$ and (b) network clustering $C_A = (1/N) \sum_i c_i$ and the clustering component $C_G = 2M/[N(N-1)]$ (see online version for colours) (continued)

Note: If $l_A < l_G$ and $C_A \gg C_G$ the graph is said to show a small-world property.

Figure 6  The relation between the numbers of nodes and links for all 21 snapshots of network topology (see online version for colours)

Note: The linear relation implies a constant mean node degree of 2.8 over the entire studied time period.
We have determined further measures such as the clustering coefficient, the edge betweenness centrality and the node information centrality. The definitions of these quantities are as follows:

- **the unweighted clustering coefficient** $c_i$ of node $i$ is calculated as:

$$ c_i = \frac{1}{n_i(n_i-1)/2} \sum_{j,m,n} a_{ij} a_{jm} a_{mn}, $$

where $n_i$ is the number of nearest neighbours of node $i$

- **the weighted betweenness centrality** of edge(s) connecting nodes $i$ and $j$ is given by:

$$ c_{ij}^B = \frac{n_{ij}}{(N-1)(N-2)n_i}, $$

where $n_{ij}$ denotes the number of weighted shortest paths that pass between nodes $i$ and $j$

- **the weighted information centrality** of node $i$ is calculated as:

$$ c_i^I = \frac{E(G) - E(G')}{E(G)}, $$

where $E(G)$ is the graph efficiency (see Equation 4) and $G'$ is a graph with $N - 1$ nodes obtained by removing the node $i$ and all its adjacent links from the original graph $G$. The graph’s efficiency is given by:

$$ E(G) = \frac{1}{N(N-1)} \sum_{i,j} \frac{1}{d_{ij}^w}. $$

Calculations were done using the network analysis tool Network Analysis Tool (NAT). The results are depicted in Figures 5 and 7. In all stages of the network evolution, more than half of the nodes has a clustering coefficient of zero. That is, these nodes are either connected in series-forming chains or as hubs of small star-shaped subgraphs. Moreover, less than 10% of the nodes have a clustering coefficient of 1. These nodes are forming small clusters, where each node in the cluster is connected to all of the other nodes. Such configurations increase the fault tolerance of the network, since the nodes remain interconnected even if some line fails. Notice that the network clustering reached its maximum in the year 1996 and some clusters were disconnected by the year 2000. The average clustering coefficient decreased from 0.2 to 0.16 in this time period (see Figure 5b). This drop can be explained by a significant increase in the number of nodes with a small clustering coefficient and a decrease in the number of nodes with a large clustering coefficient. The number of nodes with $c_i^I = 0$ between the years 1996 and 2000 increased from 110 to 131, while the number of nodes with $c_i^I > 0.5$ decreased from 70 to 61.
Figure 7  (a) The clustering coefficients $c_i^c$, (b) information centralities $c_i^I$ and (c) edge betweenness centralities $c_i^n$ for selected years (see online version for colours)
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Figure 7  (a) The clustering coefficients $c_i^c$, (b) information centralities $I_i^c$ and (c) edge betweenness centralities $B_{ij}^c$ for selected years (see online version for colours) (continued)

Note: The nodes and edges are ordered according to the measured quantity.

For information centralities $I_i^c$, we observed a positive development (see Figure 7b). The criticality of removing the most sensitive nodes is rapidly decreasing in time and the network becomes more uniform, while the outstandingly critical nodes are slowly disappearing.

The edge betweenness centralities $B_{ij}^c$ represents the relative load on links, assuming the exchange of the same load between all node pairs via the shortest paths. Looking at the betweenness centralities (see Figure 7c) and the locations of the multiple links, we found an interesting correlation. In the year 2000, for example, there are 11 double links and two triple links, i.e., 13 node pairs in total that are connected by more than one link (out of 272 node pairs directly connected by at least one link). Reordering the edges according to edge betweenness centrality in a descending order reveals that eight out of the first 13 edges are so-called multi-edges. Assuming a high-load exchange between nodes connected by multiple links, this implies a correlation between the betweenness centrality and the load. Our evaluation shows a continuous decrease in the maximal value of edge betweenness centrality until the year 1984. Thereafter, it remains approximately unchanged and the overall load distribution becomes more uniform.

4 Conclusions

The construction of an electrical transmission network is an intricate and a long process. Its contemporary topology is not the result of an exact long-term planning, but rather, the product of evolution, where many factors such as political decisions, economic success
and new technologies play an important role. In this paper, using publicly available data, we have studied the evolution of the French 400 kV transmission network over a 40-year time period, starting in the year 1960. In conclusion, we found:

- a nonlinear network growth, starting with a slow growth phase, which was followed by very intensive growth, finally reaching a phase of apparent saturation
- a constant mean node degree during the power grid construction process, while the node degree distribution changes
- a small-world property only in the saturation phase of the evolution process
- an information centrality and a betweenness centrality continuously improving with time
- an increasing clustering coefficient of the network only up to the year 1996. Later on, the clustering coefficient goes down, which may indicate a reduction of redundancy in the network. This may imply a decrease in the robustness of the power grid with respect to failures.

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References

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**Notes**

1. The French 400 kV transmission grid topological data were downloaded from the official website of the French electricity network operator RTE (http://www.rte-france.com).

2. The French demographic and electricity indicators data were purchased from the International Energy Agency (http://data.iea.org). Note that the generated power includes also the distribution losses and the own consumption of electricity producers. Therefore, the data depicted in Figure 3 do not obey the equation \(\text{generation} - \text{consumption} + \text{import} - \text{export} = 0\).

3. A similar finding was made for the road network of major German cities by Lammer et al. (2006).

4. The parallel links were substituted in the calculation by one link and its length was calculated as a parallel combination of the original links’ lengths in order to imitate the behaviour of the impedance in electrical circuits. The shortest paths, passing through the substituting link, were then uniformly distributed among the parallel links.

5. NAT is a web tool for the topological analysis of graphs. It has been realised in the framework of the EU integrated project IRRIIS. NAT can be used by registration and authentication at the website (http://irriis.nat.ylichron.it).